GAS PRESSURE AND ITS MEASUREMENT

Terminology:

Pressure (P) - the Force (F) exerted per unit Area (A) of surface

$$P = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Force (F) - the Mass (m) times the Constant Acceleration of Gravity (g)

$$F = m \times g$$

It follows:

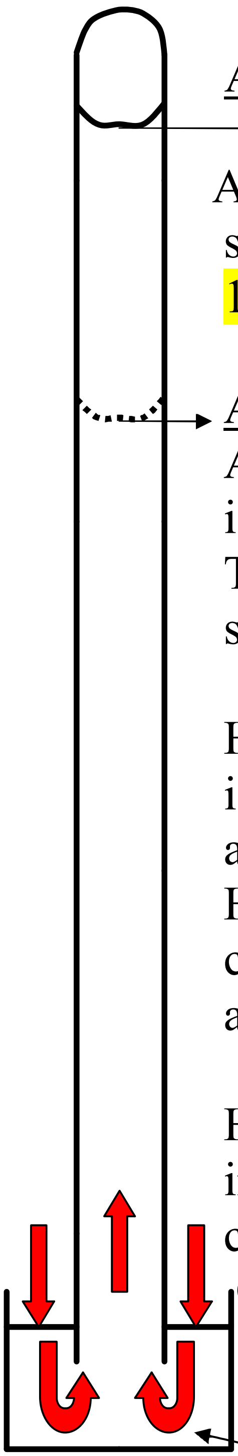
$$\mathbf{P = \frac{F}{A} = \frac{m \times g}{A} = \frac{\left\{kg\ \frac{m}{s^2}\right\}}{m^2} = \frac{kg}{m\ s^2}}$$

SI unit of Pressure is the Pascal (Pa)

$$\mathbf{1\ Pa = \frac{kg}{m\ s^2}}$$

Atmospheric Pressure:

- Atmospheric pressure is the pressure of air acting against objects in the air.
- **Air Pressure (Atmospheric Pressure)** acts in all directions (just like water pressure).
- How large is the Atmospheric Pressure?
- What is the height of the water column it can support?



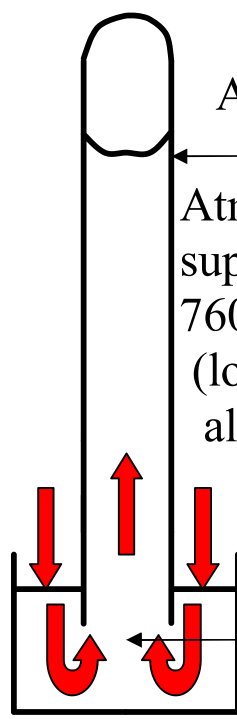
<u>At sea level</u>:
10.28 m
Atmospheric Pressure supports a water column 10.28 m high.

<u>At higher altitudes</u>:
Atmospheric Pressure is less;
The water column supported is lower.

Height of water column is proportional to atmospheric pressure;
Height of water column could be used to measure atmospheric pressure.

However, this is impractical, since the column is too high!
Could another liquid be used that results in a shorter column ?

Water (d = 1.00 g/mL)
(transmits air pressure)



At sea level
0.760 m
Atmospheric pressure supports a Hg column 760.0 mm high
(lower at higher altitudes)

Mercury
(d=13.53 g/mL)

Question:

- If atmospheric pressure supports a water column 10.298 m high, at sea level, what is the height of a mercury column it could support ?

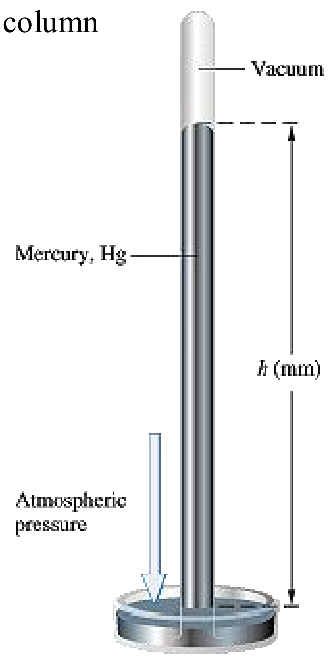
(d of water = 1.00 g/mL d of Hg = 13.53 g/mL)

Water	Mercury
H_w = 10.2 8 m	H_{Hg} = ????? (should be much less than 10.298 m)
d_w = 1.00 g/mL	d_{Hg} = 13.53 g/mL

$$H_{Hg} = 10.28 \text{ m} \times \frac{1.00 \text{ g/mL}}{13.53 \text{ g/mL}} = 0.760 \text{ m} = 760. \text{ mm}$$

The Barometer:

- It was invented by Torricelli
- It is an instrument used to measure Atmospheric Pressure
- It expresses Atmospheric Pressure in terms of the height of the mercury column supported.



Atmospheric Pressure depends on :

1. Altitude

- The higher the altitude, the lower the atmospheric pressure

2. Temperature

- The higher the temperature, the lower the atmospheric pressure

3. Humidity (moisture content of air)

- The more humid the air, the lower the atmospheric pressure

Standard Atmospheric Pressure is defined as the Atmospheric Pressure of air

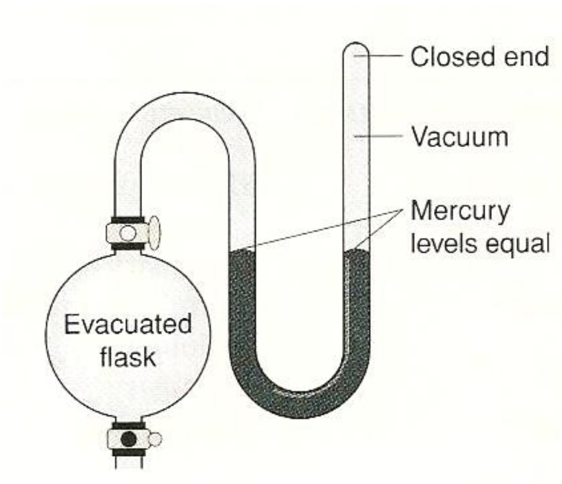
- at sea level,
- at 0^0C,
- with 0 % moisture content (100 % dry air)

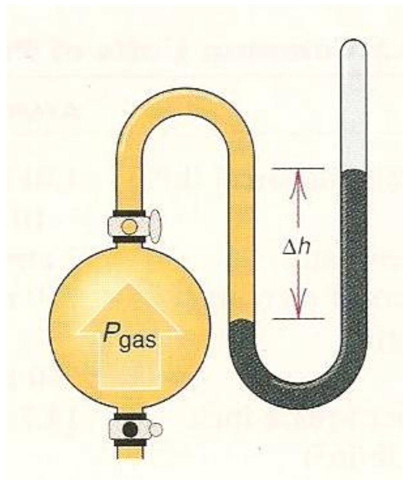
This atmospheric pressure would support a Hg column 760.0 mm high.

Standard Atmospheric pressure = 760.0 mm Hg = 760.0 torr = 1.000 atmosphere (atm)

MEASURING THE PRESSURE OF GASES IN CLOSED CONTAINERS

1. The Closed Tube Manometer

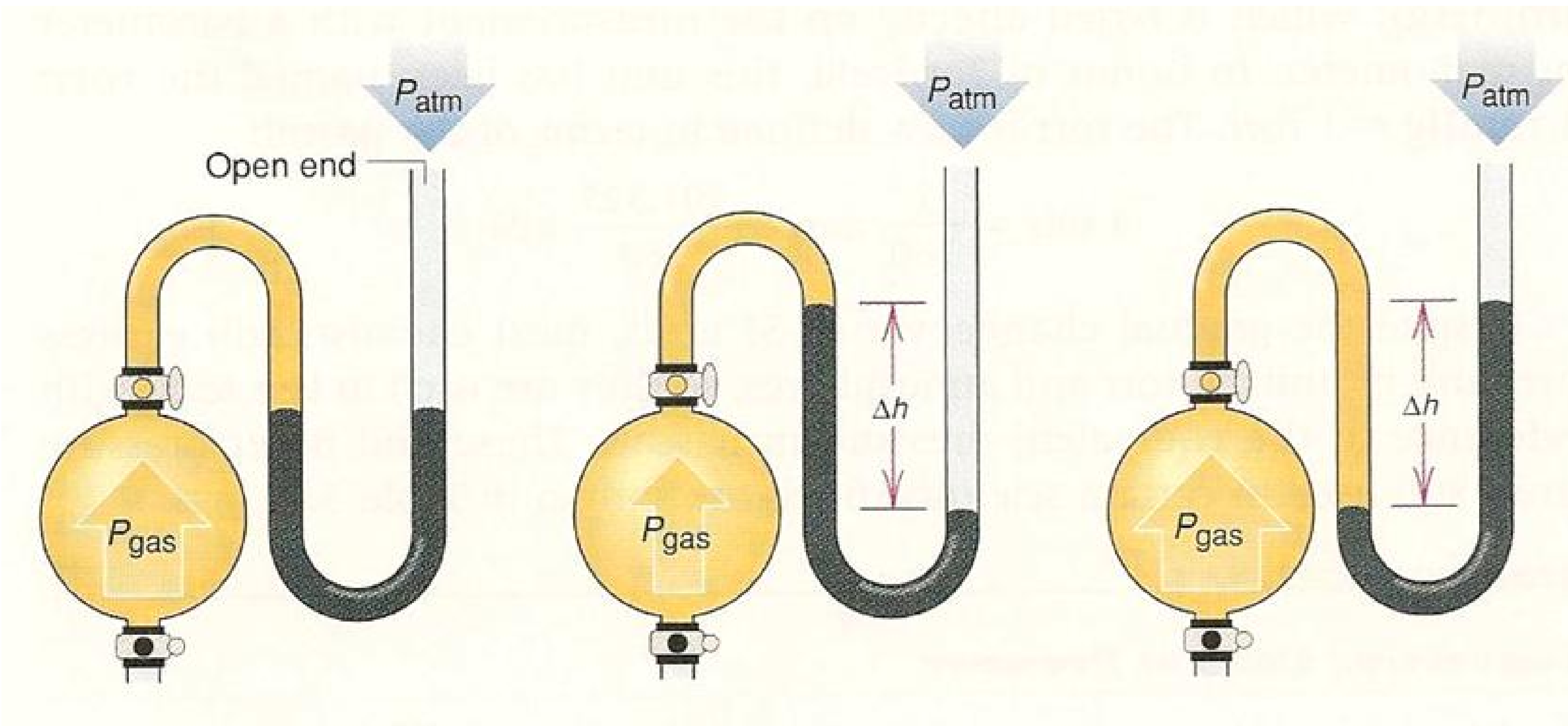




No pressure exerted

Pressure of enclosed gas = difference in height = Δh

2. <u>The Open –Tube Manometer</u>



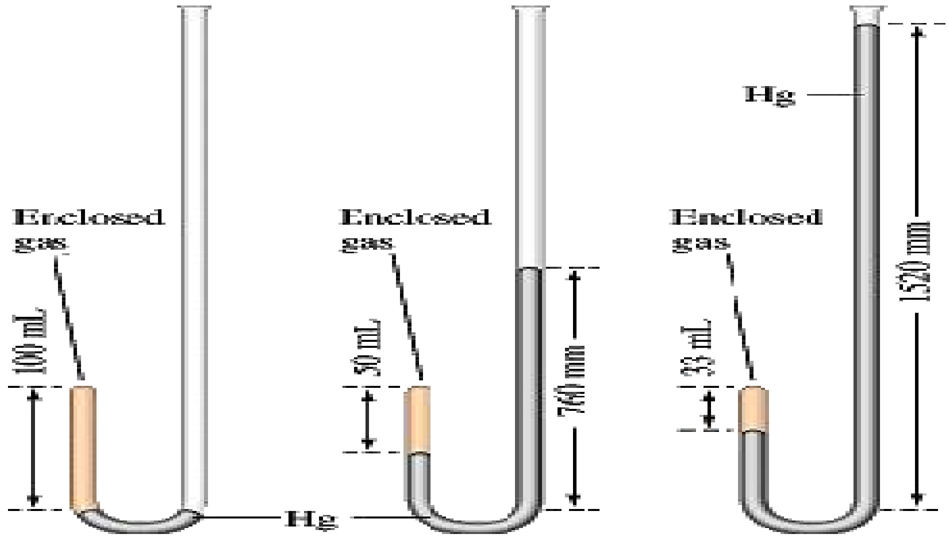
Pgas = Patm

Pgas < Patm
Pgas = Patm – Δh

Pgas > Patm
Pgas = Patm + Δh

THE GAS LAWS

- Assume the following measurements are obtained:



	Pgas = 760. mm Hg V gas = 100. mL	Pgas = 1520 mm Hg V gas = 50.0 mL	Pgas = 2280.mm Hg V gas = 33.3 mL
		• **P is doubled** • **V is halved**	• **P is tripled** • **V is one-third**
P x V	**76,000 (mmHg)(mL)**	**76,000 (mmHg)(mL)**	**76,000 (mmHg)(mL)**

NOTE:
1. Volume and Pressure are inversely proportional
2. The product of the Pressure anf the Volume is constant.

BOYLE'S LAW

- The relationship between pressure and volume is called Boyle's Law.
- **At constant temperature, the volume of a fixed amount of gas is inversely proportional to its pressure.**
- Boyle's Law can be interpreted and used both **Mathematically** and **Graphically**.

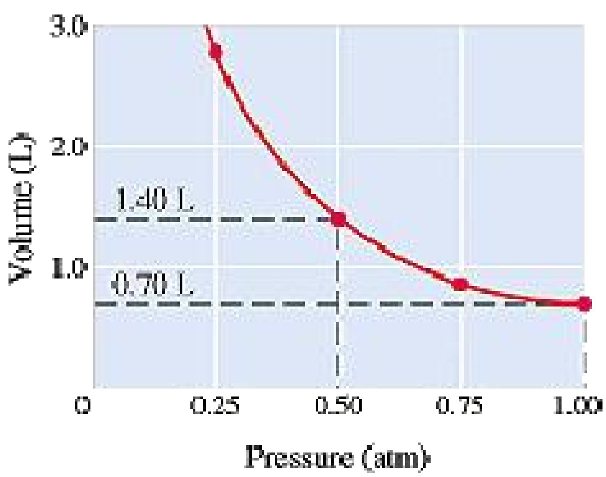
Mathematically:

PV = constant OR $\dfrac{P_1}{P_2} = \dfrac{V_2}{V_1}$ OR $\mathbf{P_1V_1 = P_2\,V_2}$

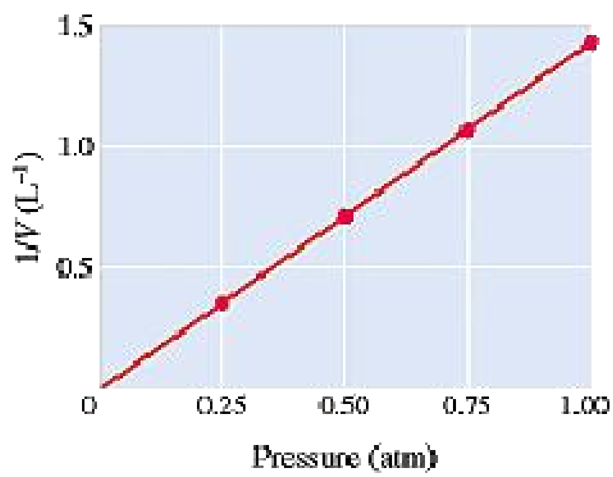
Graphically:
Assume the following data are obtained:

P (atm)	V (mL)
0.50	20.
1.0	10.
2.0	5.0

P (atm)	1/V (mL^{-1})
0.50	0.05
1.0	0.10
2.0	0.20



A



B

V is inversely proportional to P

1/V is proportional to P

Examples:

1. A gas in a closed-tube manometer has a measured pressure of 0.047 atm. Calculate the pressure in mm Hg.

$$? \text{ mm Hg} = 0.047 \text{ atm} \times \frac{760.0 \text{ mm Hg}}{1.000 \text{ atm}} = 36 \text{ mm Hg}$$

2. You have a tank or argon gas at 19.8 atm pressure at 19^0C. The volume of argon in the tank is 50.0 L. What would be the volume of this gas if you allowed it to expand to the pressure of the surrounding air (0.974 atm) ? Assume the temperature remains constant.

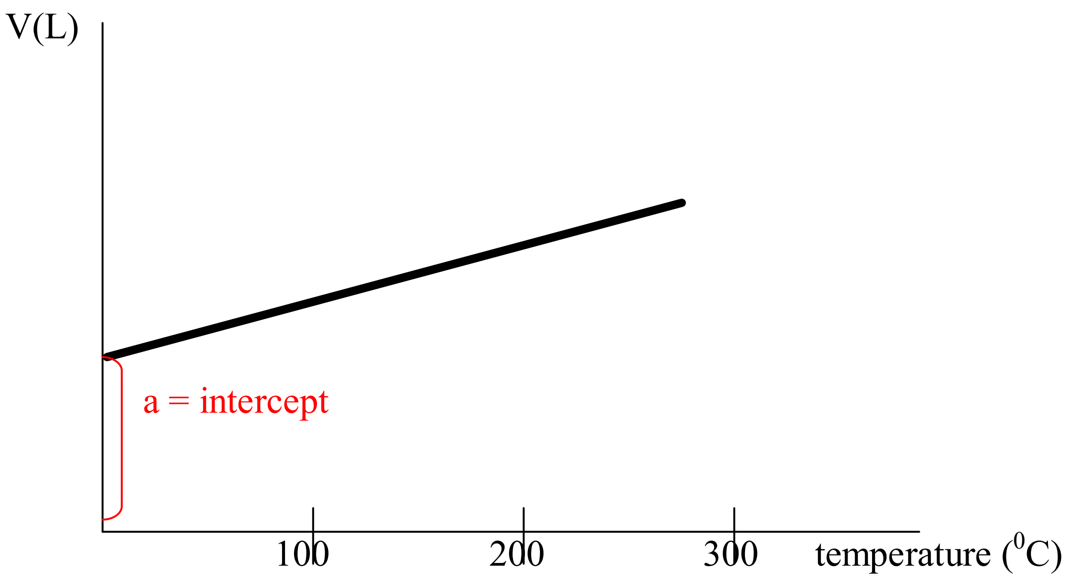
Initial	**Final**
$P_1 = 19.8$ atm	$P_2 = 0.974$ atm
$V_1 = 50.0$ L	$\mathbf{V_2 = ?}$

$$P_1 V_1 = P_2 \mathbf{V_2} \qquad V_2 = \frac{P_1 V_1}{P_2} = \frac{(19.8 \text{ atm})(50.0 \text{ L})}{(0.974 \text{ atm})} = 1020 \text{ L}$$

3. The pressure of a sample of gas with a volume of 125 mL is decreased from 2.50 atm to 1.50 atm. What is the new volume?

VOLUME-TEMPERATURE RELATIONSHIP FOR GASES

- Gases contract when cooled and expand when heated
- It follows: The Volume of a gas increases with temperature



- This is a linear relationship.
  The equation for a linear relationship is : $\mathbf{y = a + b\ x}$

$$\mathbf{V = a + b\ t}$$

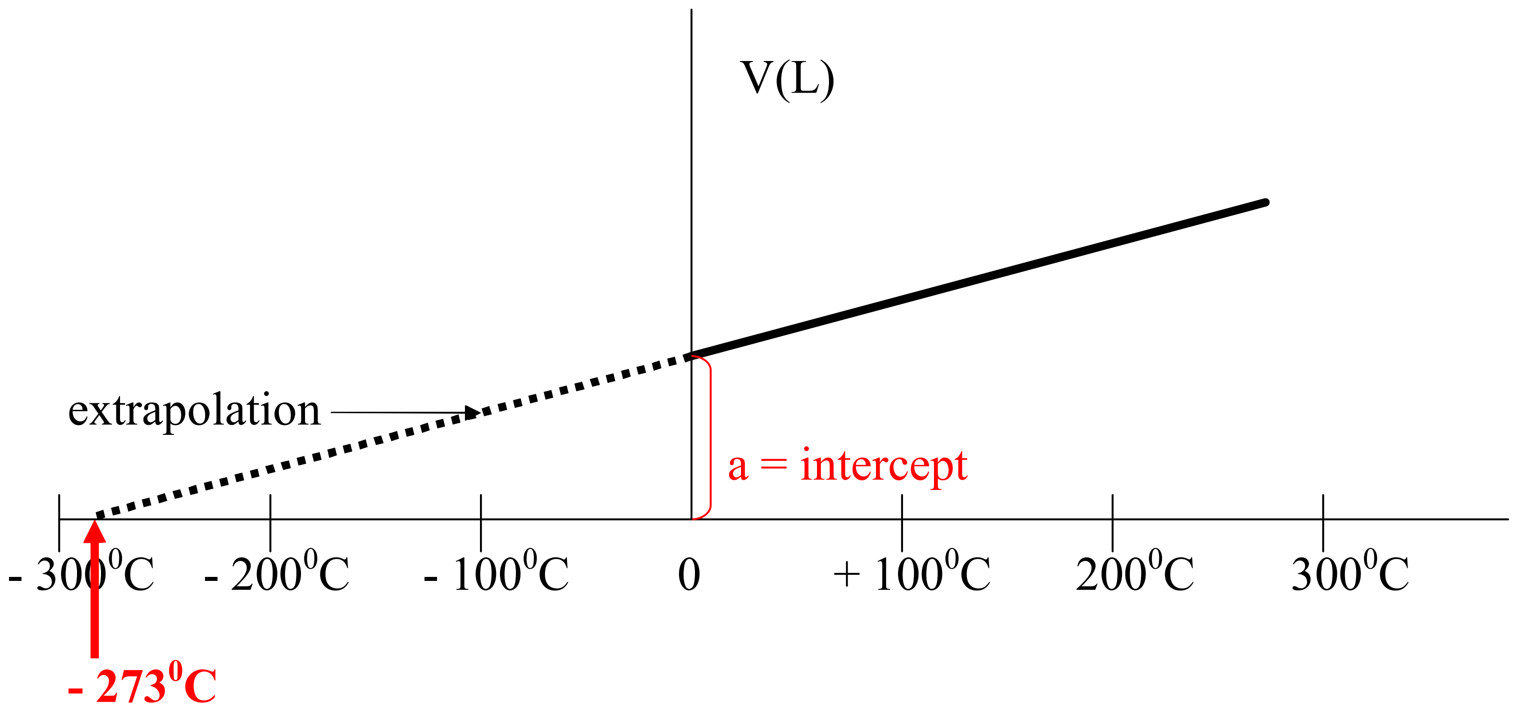
volume

intercept

slope

temperature

- To simplify the equation, the graph can be extrapolated to the temperature at which the volume of the gas becomes 0.



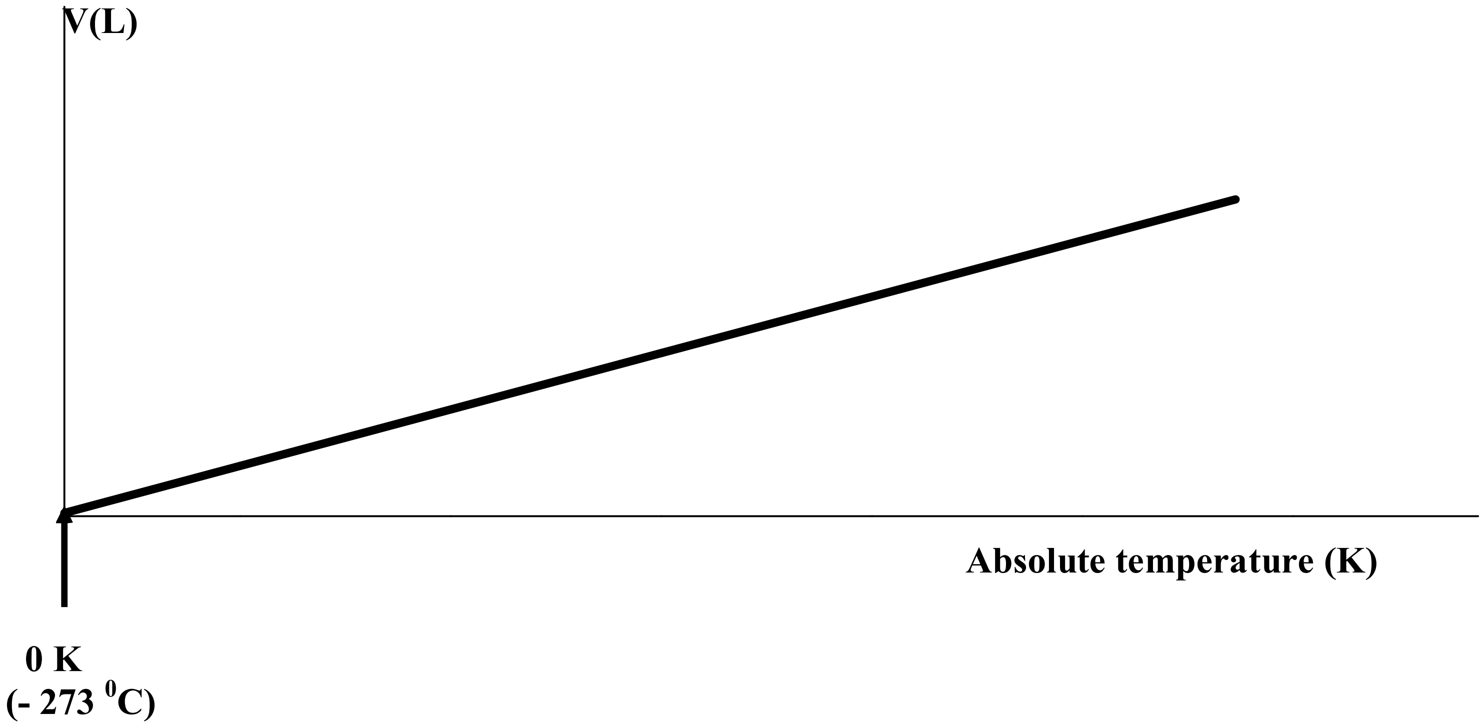
NOTE: **at** $t = -273^0C$ $V_{gas} = O$

V = a + bt becomes: $0 = a + (-273^0C)\,b$ $273\,b = a$

V = a + bt can now be rewritten: $V = 273\,b + bt = b\,(273 + t) = bT$

Absolute Temperature

- **A Plot Of Volume (V) as a function of Absolute Temperature (T)**



$$\mathbf{V = b\,T} \qquad \textbf{OR} \qquad \mathbf{\frac{V}{T} = b = constant}$$

CHARLES' LAW

- The relationship of temperature and volume in gases is called Charles' Law.
- **The Volume occupied by a gas is directly proportional to the Absolute Temperature.**

$$\boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2}} \quad \textbf{OR} \quad \boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}} \quad \textbf{OR} \quad \boxed{V_2 = V_1 \times \frac{T_2}{T_1}}$$

<u>Examples:</u>

1. An experiment calls for 5.83 L of sulfur dioxide gas (SO_2) at 0 °C and 1.00 atmospheres. What would be the volume of this gas at 25 °C and 1.00 atm ? (Note: the pressure does not change)

<u>Initial</u>	**<u>Final</u>**	
t_1 = 0 °C	t_2 = 25 °C	
T_1 = 0 °C + 273 = 273 K	T_2 = 25 °C + 273 = 298 K	Temperature increases
V_1 = 5.83 L	**V_2 = ?**	**Volume must increase**

$$V_2 = 5.83\text{ L} \times \frac{298\text{ K}}{273\text{ K}} = \mathbf{6.36\text{ L}}$$

OR

$$V_2 = \frac{V_1\, T_2}{T_1} = \frac{(5.83\text{ L})\,(298\text{ K})}{(273\text{ K})} = \mathbf{6.36\text{ L}}$$

2. A sample of nitrogen gas with a volume of 10.0 L has a temperature of –78°C. What is the volume of this gas at 25°C at constant pressure?

Combined Gas Law

- Boyle's Law and Charles Law can be combined and expressed in a single statement.
- The Volume of a gas:

- is inversely proportional to its Pressure → **Boyle's Law** → $V \propto \frac{1}{P}$

- is directly proportional to its Absolute Temperature → **Charles' Law** → $V \propto T$

- For a given amount of gas, this can be written as a single equation:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

T = ct

When temperature is constant: $(T_1 = T_2 = T)$

$$\frac{P_1V_1}{\cancel{T}} = \frac{P_2V_2}{\cancel{T}}$$

$$P_1V_1 = P_2V_2$$

Boyle's law

P = ct

When pressure is constant : $(P_1 = P_2 = P)$

$$\frac{\cancel{P}\,V_1}{T_1} = \frac{\cancel{P}\,V_2}{T_2}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Charles' Law

Alternately:

$$V_2 = V_1 \times \frac{P_1}{P_2} \times \frac{T_2}{T_1}$$

Examples:

1. A bacterial culture isolated from sewage produced 41.3 mL of methane gas (CH_4) at 31 °C and 753 mm Hg pressure. What is the volume of the methane gas at 0 °C and 760.0 mm Hg ?

Initial	**Final**
$V_1 = 41.3$ mL	$V_2 = ?$
$t_1 = 31$ °C	$t_2 = 0$ °C
$T_1 = 304$ K	$T_2 = 273$ K
$P_1 = 753$ mm Hg	$P_2 = 760.0$ mm Hg

$V_2 = V_1$ x Temperature Ratio x Pressure Ratio

T↓ V↓ P↑ V↓

$$V_2 = 41.3\text{ mL} \times \frac{273\text{ K}}{304\text{ K}} \times \frac{753\text{ mmHg}}{760.0\text{ mm Hg}} = \mathbf{36.7\text{ mL}}$$

OR

$$V_2 = \frac{P_1 V_1 T_2}{T_1 P_2} = \frac{(753\text{ mmHg})(41.3\text{ mL})(273\text{ K})}{(304\text{ K})(760.0\text{ mm Hg})} = \mathbf{36.7\text{ mL}}$$

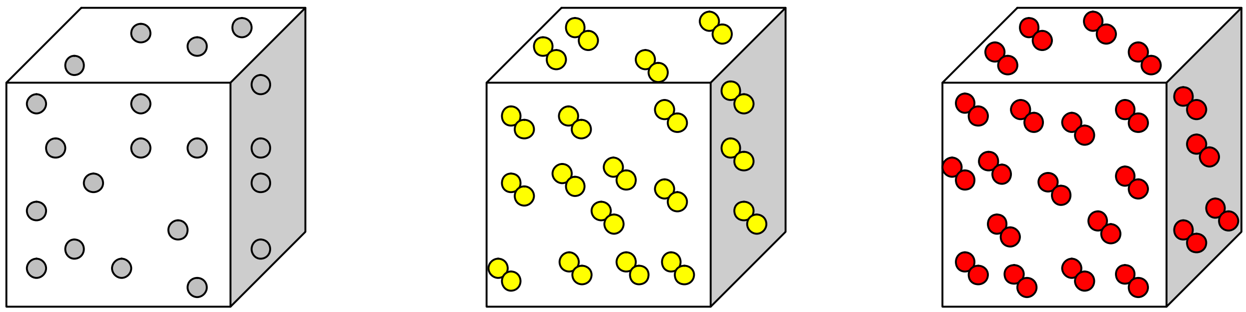
2. The pressure of 1.50 L of a gas is doubled and its absolute temperature is increased 3 times. What will the final volume of the gas be?

3. A sample of Cl_2 occupies 8.50 L at 80 °C and 745 mmHg. What volume will it occupy at 30°C and 1.20 atm?

RELATIONSHIP BETWEEN VOLUME AND AMOUNT OF GAS

NOTE:
- Volume of gas is expressed in L or mL (V)
- Amount of gas is expressed in number of moles of gas (n)

- Consider equal volumes of three different gases at the same temperature and pressure:



1 L He   1 L H_2 1L O_2

- **Same Volume, Same temperature, Same Pressure**
- Which cube contains the largest number of molecules ?

- **The three cubes contain the same number of molecules!**

Reasons:

1. The volume of a gas is determined by the intermolecular distances;

   Intermolecular distances: - depend on Volume and Pressure (the same)
   - do not depend on the type of gas

2. The volume occupied by the molecules is not the same in the three samples of gas.

   However, the volume occupied by the molecules is negligible compared to the intermolecular distances (molecules of gas are very far apart from each other)

AVOGADRO'S LAW

- **Equal volumes of different gases at the same temperature and pressure, contain the same number of molecules.**

Consequences:

1. Equal number of different gaseous molecules (equal number of moles of gas), at the same temperature and pressure, occupy equal volumes.

2. 1 mole of any gas (6.02×10^{23} gaseous molecules) occupies the same volume under the same conditions of temperature and pressure.

At 0^oC and 760.0 torr (1.000 atm), **1 mol** of any gas **occupies 22.4 L**

V_m = Molar Gas Volume = 22.4 L

0^0C(273 K) and 760.0torr(760.0mm Hg, or 1.000 atm) are referred to as:
STP (Standard Temperature and Pressure)

V_m = 22.4 L at STP (O^0C and 760.0 torr)

3. The Volume of a gas is directly proportional to the number of gaseous molecules it contains (number of moles of gas)

V = Volume of gas
n = Number of moles of gas

$$\underbrace{V \propto n}_{\text{directly proportional}} \qquad \text{or} \qquad \underbrace{V = a\,n}_{\text{constant}}$$

Examples:

1. 1 mol of gas occupies 22.4 L at STP. What is the volume at 20.0^{0}C and 749 mm Hg ?

Initial	**Final**
V_1 = 22.4 L	**V_2 = ?**
T_1 = 0^{0}C	t_2 = 20.0^{0}C
T_1 = 273 K	T_2 = 293 K
P_1 = 760.0 mm Hg	P_2 = 749 mm Hg

V_2 = V_1 x Temperature Ratio x Pressure Ratio

T↑ V↑ P↓ V↑

$$V_f = 22.4 \text{ L} \times \frac{293 \text{ K}}{273 \text{ K}} \times \frac{760.0 \text{ mmHg}}{749 \text{ mm Hg}} = \mathbf{24.4\ L}$$

2. A sample of oxygen gas with a mass of 46.0 g is contained in a 175 mL flexible container at a particular temperature and pressure. If 10.0 g of the gas leaks out, what is the volume of the container at the same temperature and pressure?

3. A propane (C_3H_8) tank is opened and 30.0 L of gas at STP is released. What is the mass of the gas released?

The Ideal Gas Law

- The 3 relationships that have been derived for gases can be combined into a single equation.

1. The **Volume** of a gas is **inversely proportional** to its **Pressure**

$$\mathbf{V = \underset{\substack{\uparrow \\ \text{Constant}}}{k} \frac{1}{P}}$$

Boyle's Law

2. The **Volume** of a gas is **directly proportional** to its **Absolute Temperature**

$$\mathbf{V = \underset{\substack{\uparrow \\ \text{Constant}}}{b}\ T}$$

Charles' Law

3. The Volume of a gas is directly proportional to the number of moles, n

$$\mathbf{V = \underset{\substack{\uparrow \\ \text{Constant}}}{a}\ n}$$

Avogadro's Law

It follows: $V \propto \frac{T\,n}{P}$ ➡ $V = \underset{\substack{\uparrow \\ \text{Molar Gas Constant} \\ \text{(Proportionality Constant)}}}{R} \frac{T\,n}{P}$ ➡ **P V = n R T Ideal Gas Equation**

The Ideal Gas Law

- To determine the value of R (Molar Gas Constant), consider exactly 1 mol of gas at STP.

P = 1.00 atm
T = 273 K
n = 1.00 mol
V = 22.4 L
R = ?

$$P\,V = n\,R\,T \qquad R = \frac{P\,V}{n\,T} = \frac{(1.00\text{ atm})\,(22.4\text{ L})}{(1.00\text{ mol})\,(273\text{ K})}$$

$$\boxed{R = 0.0821\,\frac{\text{L . atm}}{\text{K . mol}}}$$

<u>Examples:</u>

1. What is the pressure in a 50.0 L tank that contains 3.03 kg of oxygen gas at 23^0C ?

V = 50.0 L

$$n = 3{,}0\underline{3}0\text{ g} \times \frac{1\text{ mol } O_2}{32.00\text{ g } O_2} = 94.\underline{6}9\text{ mol } O_2$$

$T = 23^0C + 273 = 296$ K

$$R = 0.0821\,\frac{\text{L . atm}}{\text{K . mol}}$$

P = ?

$$P\,V = n\,R\,T$$

$$P = \frac{n\,R\,T}{V}$$

$$P = \frac{(94.\underline{6}9\ \cancel{\text{mol}})(0.0821\,\frac{\cancel{\text{L}}\ \text{atm}}{\cancel{\text{K}}\ \cancel{\text{mol}}})(296\ \cancel{\text{K}})}{50.0\ \cancel{\text{L}}} = 46.0\text{ atm}$$

Examples:

2. What is the density of carbon dioxide gas (in g/L) at STP ?

$$d = \frac{m}{V}$$

For 1 mol of CO_2, at STP:

$m = 12.01\ g + 32.00\ g = 44.01\ g$

$V = 22.4\ L$

$$d = \frac{44.01\ g}{22.4\ L} = 1.96\ g/L$$

3. What is the **density** of carbon dioxide gas (in g/L) at 22^0C and 751 mm Hg?

$$\mathbf{d} = \frac{m}{\mathbf{V}}$$

For 1 mol of CO_2, at 22^0C and 751 mm Hg:

$m = 12.01\ g + 32.00\ g = 44.01\ g$

V = ? L

$P\,\mathbf{V} = n\,R\,T \qquad \mathbf{V} = \frac{n\,R\,T}{P}$

$$\mathbf{d} = \frac{\mathbf{44.01\ g}}{\mathbf{24.5\underline{1}\ L}}$$

$$\mathbf{V} = \frac{(1.00\ \cancel{mol})(0.0821\ \frac{L\ \cancel{atm}}{\cancel{K}\ \cancel{mol}})(295\ \cancel{K})}{(751\ \cancel{mmHg}\ x\ \frac{1.000\ \cancel{atm}}{760.0\ \cancel{mmHg}}} = \mathbf{24.5\underline{1}\ L}$$

d = 1.80 g/L

Examples:

4. The density of a gas at 90.^{0}C and 753 mm Hg is 1.585 g/L. What is the **Molecular Weight** of the gas?

$$d = \frac{1.585\ g}{1.000\ L}$$

implies

m = 1.585 g

V = 1.000 L

P = 753 mm Hg
T = 363 K

$$? \frac{g}{mol}$$

n = ?

$$P V = n R T \qquad n = \frac{P V}{R T}$$

$$n = \frac{753\ \cancel{mmHg}\ \left(\frac{1\ \cancel{atm}}{760\ \cancel{mmHg}}\right) \times 1.000\ \cancel{L}}{\left(0.0821 \frac{\cancel{L}\ \cancel{atm}}{\cancel{K}\ mol}\right) \times 363\ \cancel{K}} = 0.033\underline{2}5\ mol$$

$$\text{Molecular Weight} = \frac{1.585\ g}{0.033\underline{2}5\ moles} = 47.7\ g/mol$$

STOICHIOMETRY PROBLEMS INVOLVING GAS VOLUMES

- When doing stoichiometry calculations with gases as reactants or products, use volume, temperature and pressure data to obtain moles of gas using the ideal gas law.
- Subsequently use molar ratios in a balanced chemical equation to perform stoichiometric calculations as done previously.

Examples:

1. Magnesium reacts with hydrochloric acid to produce magnesium chloride and hydrogen gas. Calculate the volume of hydrogen (in L), produced at 28^{0}C and 665 mmHg, from 0.0420 mol magnesium and excess hydrochloric acid.

$$\underset{0.0420\text{ mol}}{Mg(s)} + 2\,HCl(aq) \longrightarrow MgCl_2(aq) + \underset{\substack{?\text{ moles}\\ ?\text{ L}}}{H_2(g)}$$

Part 1: Stoichiometry

$$?\text{ moles } H_2 = 0.0420\text{ mol Mg} \times \frac{1\text{ mol } H_2}{1\text{ mol Mg}} = 0.0420\text{ mol } H_2$$

Part 2: Ideal Gas Law

n = 0.0420 mol H_2
T = 301 K
P = 665 mm Hg
V = ?

$$pV = nRT \qquad V = \frac{nRT}{P}$$

$$V = \frac{(0.0420\ \cancel{\text{mol}})(0.0821\ \frac{\text{L}\ \cancel{\text{atm}}}{\cancel{\text{K}}\ \cancel{\text{mol}}})(301\ \cancel{\text{K}})}{665\ \cancel{\text{mmHg}} \times \frac{1\ \cancel{\text{atm}}}{760\ \cancel{\text{mmHg}}}} = 1.19\text{ L}$$

2. Calculate the volume of CO_2 at STP that can be formed from the decomposition of 152 g of $CaCO_3$, as shown below.

$$CaCO_3\ (s) \rightarrow CaO\ (s) + CO_2\ (g)$$

3. What volume of oxygen gas at 27°C and 746 mmHg would be generated by decomposition of 125 g of a 50.0% by mass solution of hydrogen peroxide, as shown below? Ignore any water vapor present.

$$2\ H_2O_2\ (aq) \rightarrow 2\ H_2O\ (l) + O_2\ (g)$$